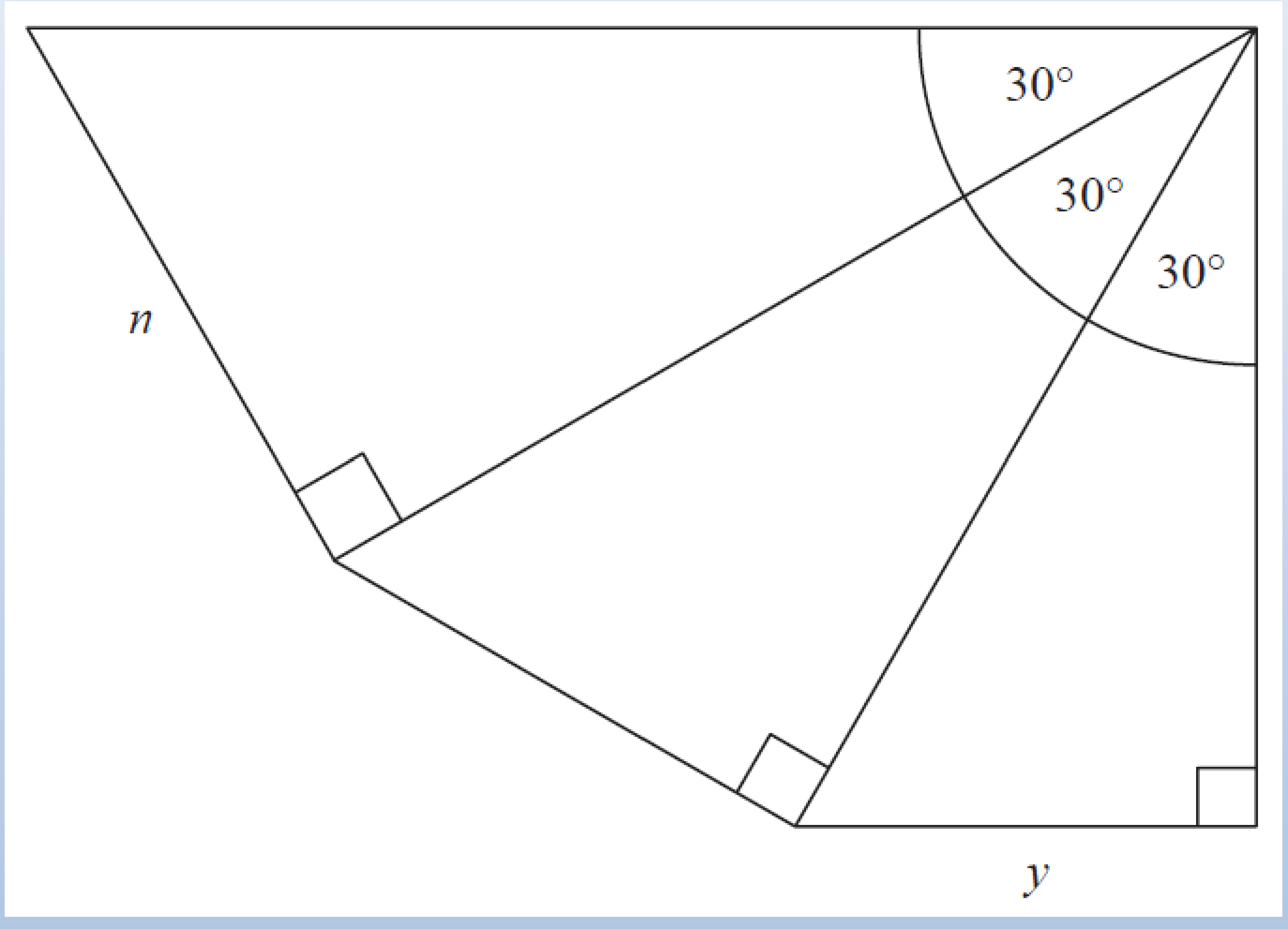
Starter: non-calculator

Prove that y = ¾ n



Starter: non-calculator

B1	for using any correct trig value for 30°, e.g. $\sin 30 = 0.5$, $\cos 30 = \dfrac{\sqrt{3}}{2}$ or $\tan 30 = \dfrac{1}{\sqrt{3}}$
M1	for hypotenuse of small triangle = $2y$ or hypotenuse of large triangle = $2n$
A1	for method to find the hypotenuse of middle triangle, e.g. $\sqrt{(2n)^2 - n^2}$ $(= \sqrt{3}n)$
A1	for a correct equation linking y and n and correct working leading to the given result

E Trigonometry

E1

Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$

Work with radian measure, including use for arc length and area of sector.

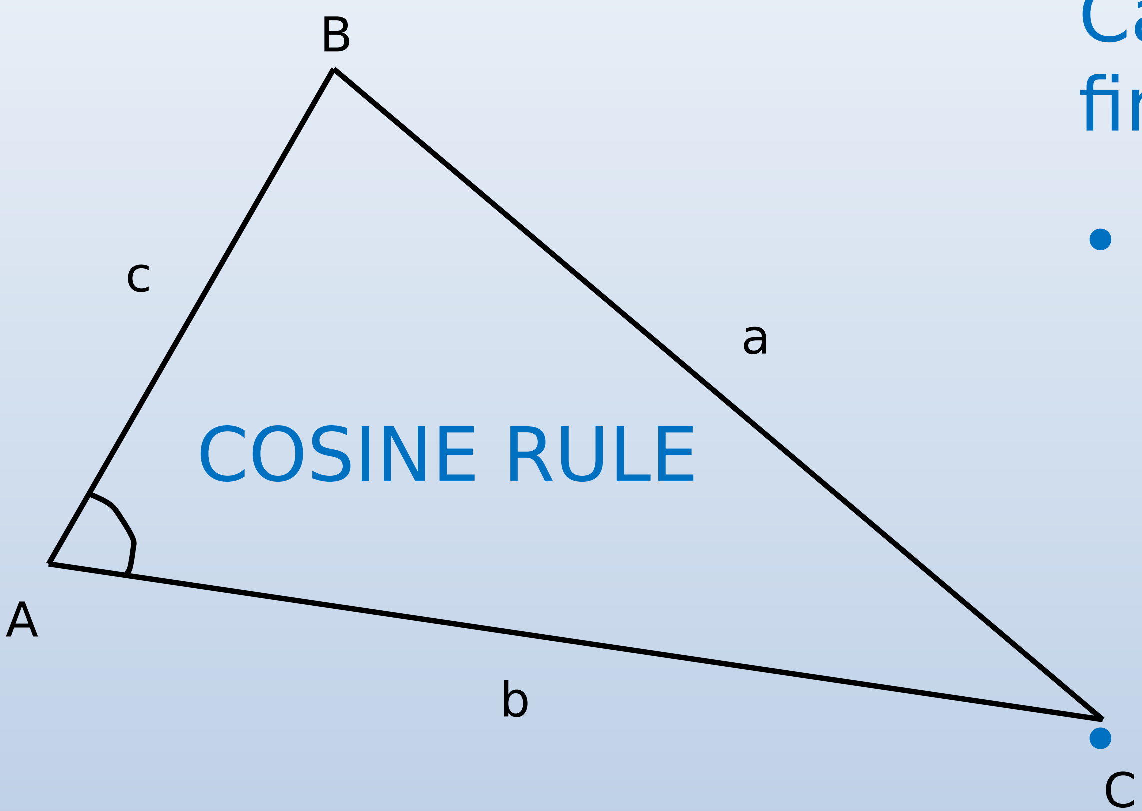
Students should:

- know and be able to apply the following rules:

  In any triangle ABC

  - area of triangle $\frac{1}{2}ab\sin C$
  - sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
  - cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$
- be aware of the ambiguous case that can arise from the use of the sine rule.

3.2 Sine and cosine rules



Can be used to find..

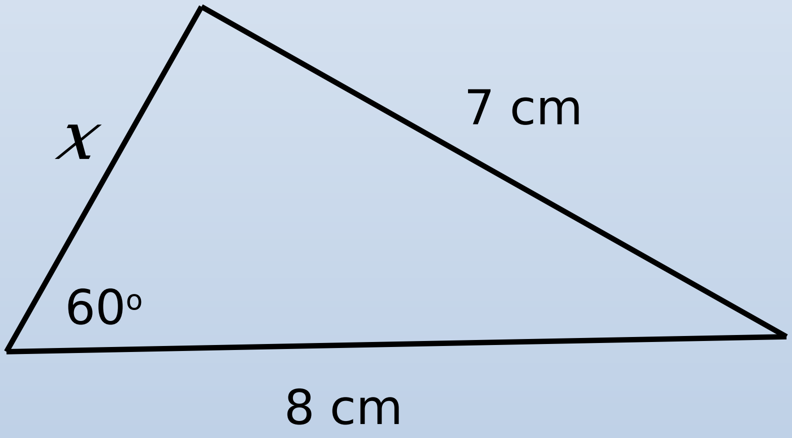
- The THIRD SIDE when you know two sides and the angle between them
- AN ANGLE when you know the three sides

$$a^2 = b^2 + c^2 - 2bc\cos A$$

3.2 Sine and cosine rules

Example 1

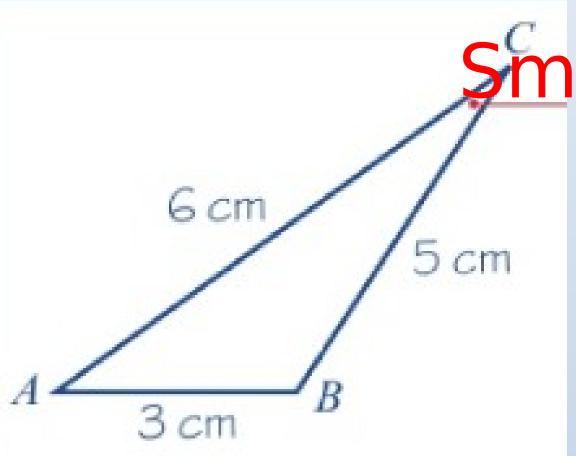
Find the possible lengths of the side marked in this triangle.



3.2 Sine and cosine rules

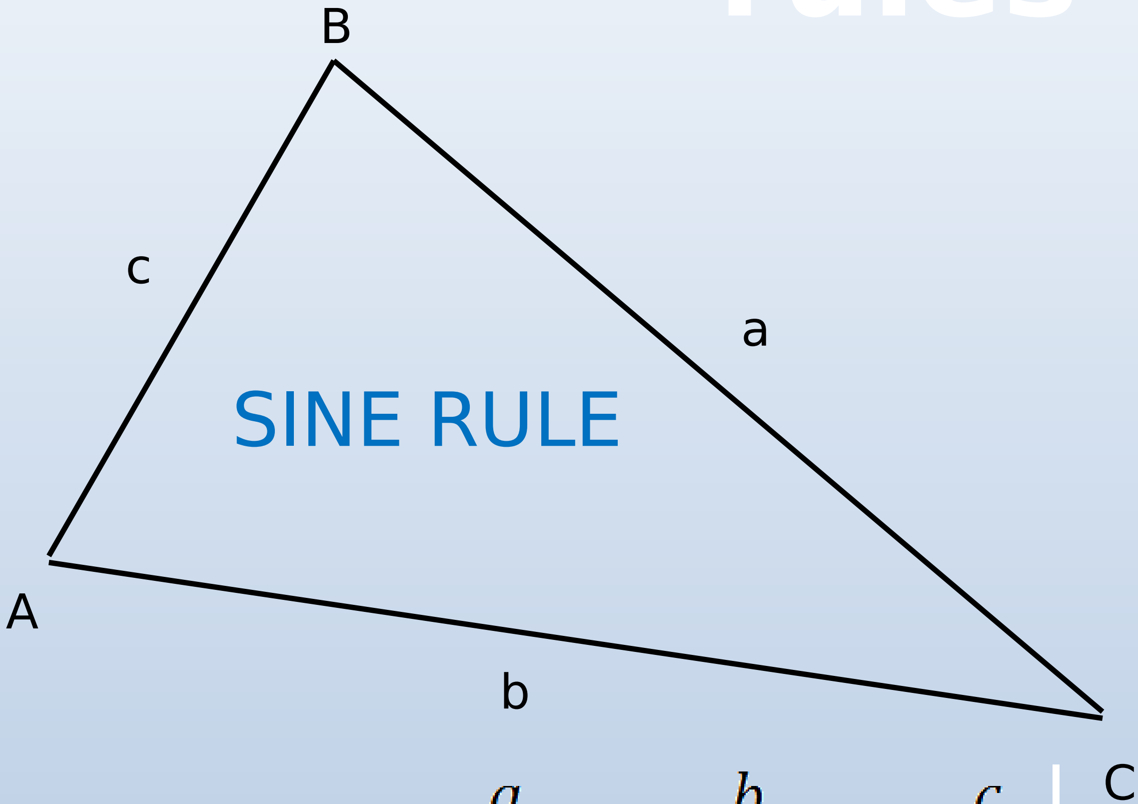
Example 2

Find the size of the smallest angle in a triangle whose sides have lengths 3 cm, 5 cm and 6 cm.



Smallest angle is opposite the shortest

3.2 Sine and cosine rules



SINE RULE

Can be used to find..

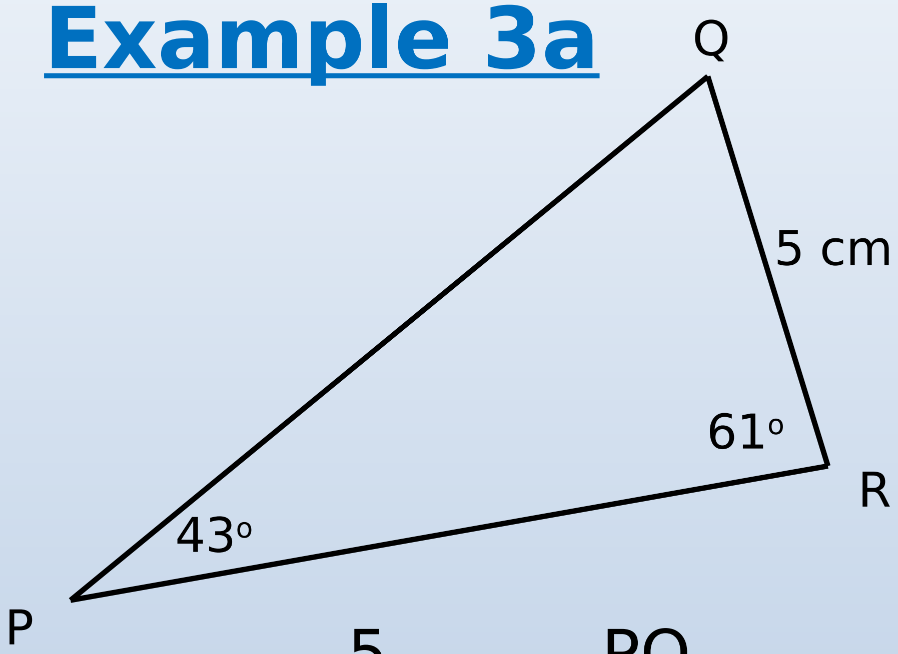
- A SIDE when you know the angles in the triangle and a side
- AN ANGLE when you know two sides and the angle

To find a length $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

To find an angle $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

3.2 Sine and cosine rules

Example 3a



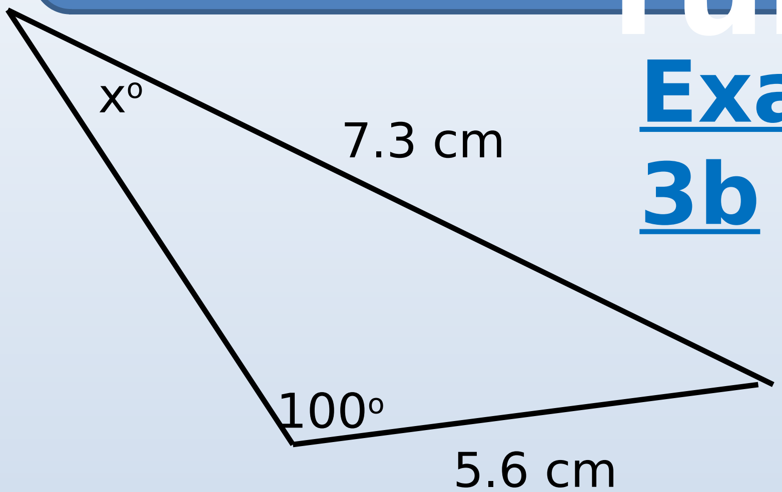
Find length PQ.
Give your answer to 3sf.

$$\frac{5}{\sin 43^\circ} = \frac{PQ}{\sin 61^\circ}$$

$$\frac{5}{\sin 43^\circ} \times \sin 61^\circ = PQ$$

$$6.41 = PQ \text{ (to 3sf)}$$

3.2 Sine and cosine rules



Example 3b

Find the labelled angle.
Give your answer to 3sf.

$$\frac{\sin x^\circ}{5.6} = \frac{\sin 100^\circ}{7.3}$$

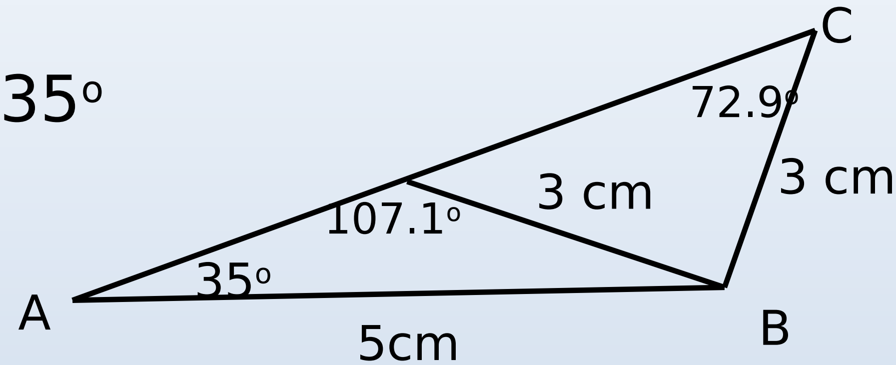
$$\sin x = \frac{\sin 100^\circ}{7.3} \times 5.6$$

$$x = \sin^{-1}\left(\frac{\sin 100^\circ}{7.3} \times 5.6\right)$$

$$x = 49.1^\circ \text{ (to 3sf)}$$

mple 4 - The Ambiguous Case of the Sine Rule!

In Triangle ABC, Find angle C given that AB = 5cm, BC = 3cm and
angle A = 35°



$$\frac{\sin C}{5} = \frac{\sin 35^\circ}{3}$$

$$\sin C = \frac{\sin 35^\circ}{3} \times 5$$

$$C = \sin^{-1}\left[\frac{\sin 35^\circ}{3} \times 5\right] = 72.9 \text{ (to 1dp)}$$

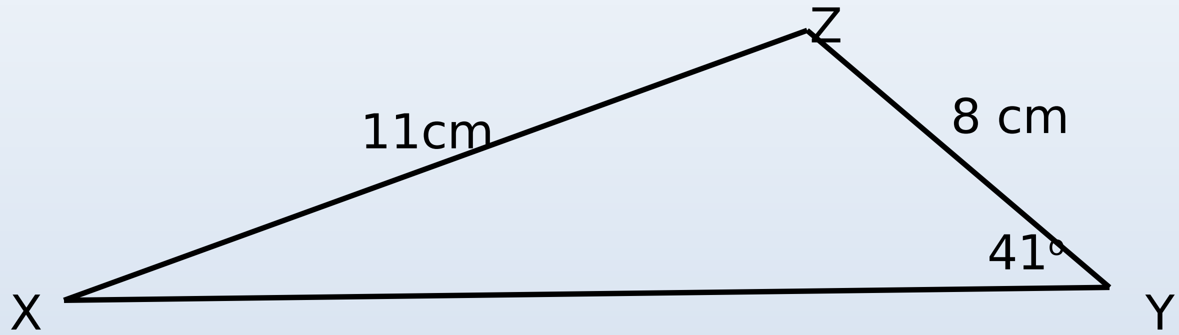
You MUST check for the AMBIGUOUS CASE when you have 2 sides and a non-included angle.

owever there is another angle with the same sine value...107.1°

107.1° + 35° = 142.1°. Angle B will be 37.9° in this case.
So a second triangle is possible.

this triangle ambiguous ?

Triangle XYZ, $<Y = 41^o$, XZ = 11cm and YZ = 8cm. Find $<X$



$$\frac{\sin X}{8} = \frac{\sin 41^o}{11}$$

$$\sin X = \frac{\sin 41^o}{11} \times 8$$

$$x = \sin^{-1}\left[\frac{\sin 41^o}{11} \times 8\right] = 28.5 \text{ (to 1dp)}$$

he obtuse angle with the same sine value..151.5°

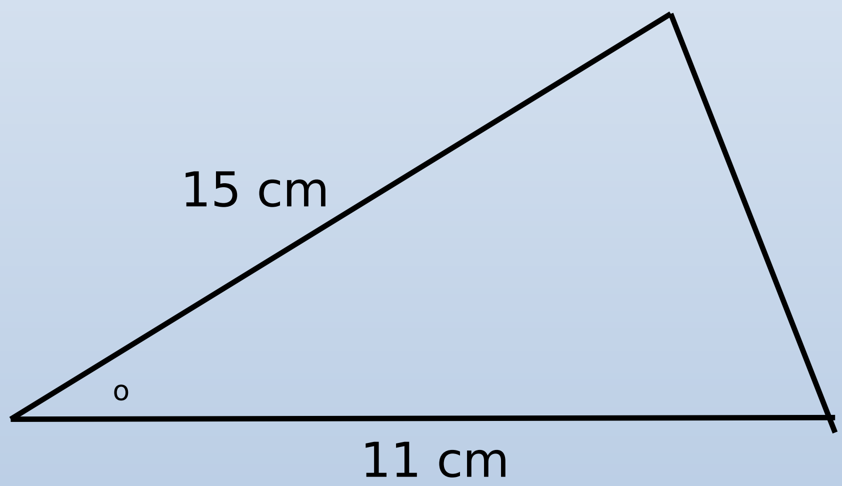
1.5° + 41° = 192.5°; (since >180°, a second triangle is not possible.)

3.2 Sine and cosine rules

Example 5

$$A = \frac{1}{2} ab\,SinC$$

The area of the triangle is . Find the two possible values of

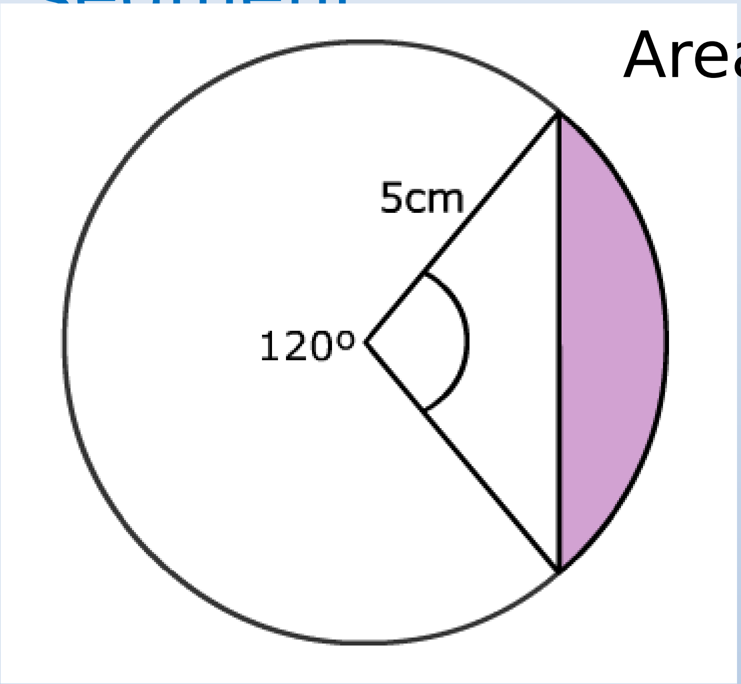


Or

3.2 Sine and cosine rules

Example 6

Calculate the area of the purple shaded segment



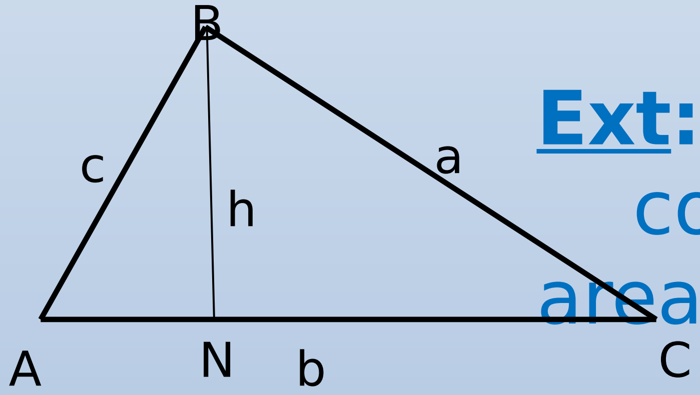
Area of triangle:

Area of sector:

purple segment:

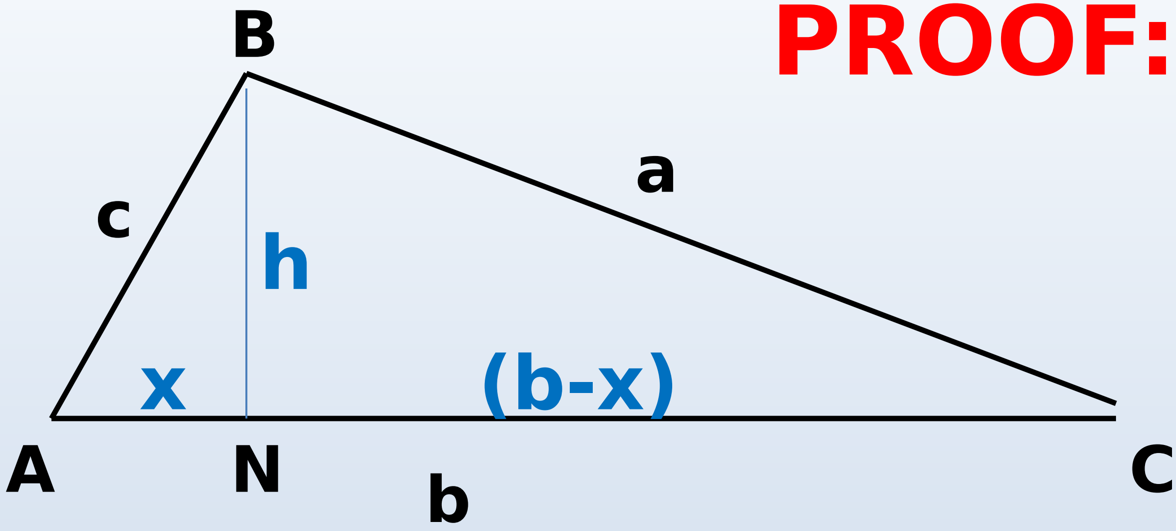
3.2 Sine and cosine rules

W/sheet:
Cambridge
AQA Mixed
Practice 11



Ext: Can you prove the cosine rule and the area rule?

PROOF: COSINE RU



1. Using Pythagoras in triangle ANB:

2. Using Pythagoras in triangle BNC:

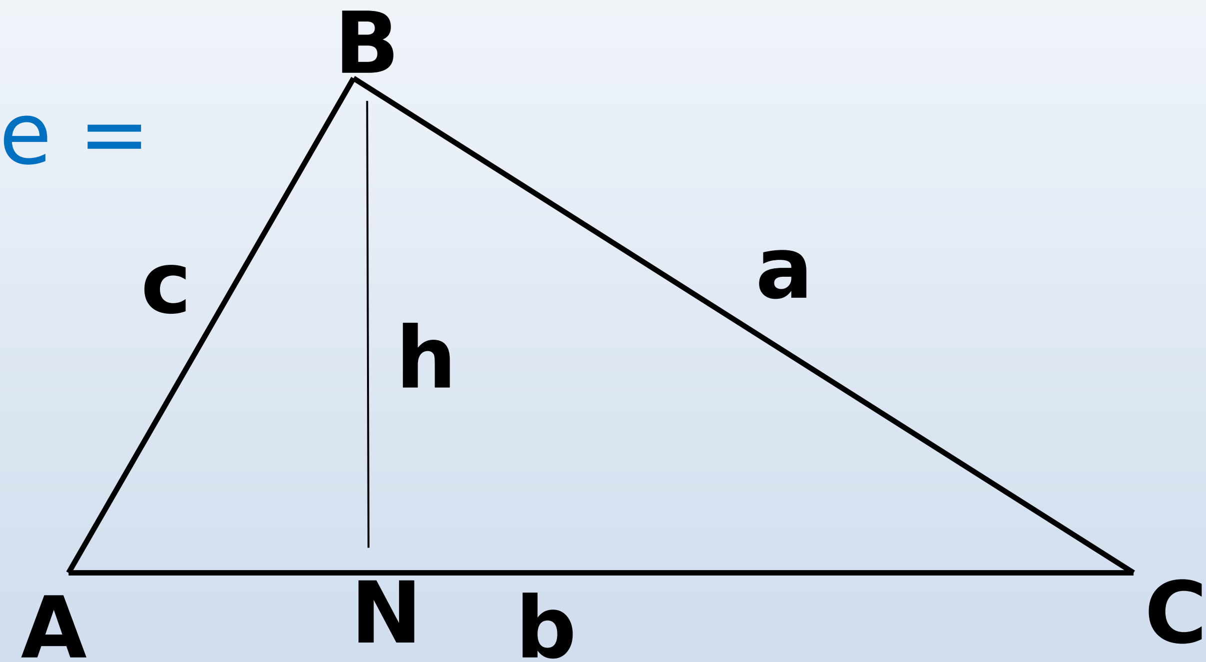
3. Substitute into 2 □

4. In triangle ANB:

5. Substitute into 3 □

OOF: area of a triangle

Area of triangle =



In triangle BCN: $\sin C = \dfrac{h}{a}$ so

So, area of triangle ABC

C is the INCLUDED angle between sides a and b